Sovereign Debt and the Effects of Fiscal Austerity*

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LATEST VERSION

Abstract

I study the impact of austerity programs implemented in the Eurozone since 2010. To do so I incorporate strategic sovereign default into a DSGE model where the government follows fiscal rules, which are estimated from data. I calibrate the model using data from Spain and estimate the size and impact of fiscal policy shocks associated with austerity policies. I then use the model to predict what would have happened to output, consumption, employment, sovereign debt levels and spreads if Spain had continued to follow the pre-2010 fiscal rule instead of switching to the austerity track. I find that, contrary to the expectations of policy makers at the time, austerity increased rather than decreased sovereign spreads and did not reduce debt to GDP ratios. Furthermore it had a negative impact on employment and GDP. Austerity was self-defeating.

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1 Introduction

The European debt crisis of 2010 triggered a debate on the potential effects of fiscal austerity. On the one hand, some economists claim that austerity is beneficial because it can increase creditworthiness and reduce sovereign spreads.\footnote{\textit{"Sharp corrections are needed in countries that already face high and increasing risk premia on their debt. Failure to consolidate would not only raise the cost of borrowing for the government; it would also undermine macroeconomic stability with widespread economic costs."} Corsetti (2010)} On the other, those against fiscal consolidation argue that it can worsen recessions and even become self-defeating or ineffective in reducing debt to GDP ratios and default risk.\footnote{See Corsetti (2012) for a summary of the discussion. Also see Cafiso and Cellini (2012), Cottarelli (2012), Gros (2011), Krugman (2015) and Wren-Lewis (2016).}

This policy debate started when most Eurozone countries were implementing fiscal austerity packages. Indeed, most of these policies were carried out after 2010, at the beginning of the European debt crisis. Hence, one way to contribute to this debate is to assess the ex post consequences of these policy measures. This paper aims to do so by taking one country, Spain, and asking: What was the impact of fiscal austerity from 2010 to 2014? In particular I focus on the effects on real variables (i.e., GDP, consumption, employment) and fiscal variables such as sovereign spreads and debt to GDP ratios.

To address this question, I propose a small open economy Dynamic Stochastic General Equilibrium (DSGE) model that incorporates the trade-off behind the debate. In this model, fiscal austerity can reduce debt and sovereign spreads, but it can also cause a deeper recession with higher unemployment that might in turn increase default risk and spreads. The model has three salient characteristics. First, in order to display a realistic fiscal policy it incorporates fiscal rules estimated using historical data up to 2007. These rules enter the model in a similar way to how a Taylor rule is incorporated into a nominal DSGE model. The main reason for using estimated fiscal rules instead of letting the government optimally choose government spending and taxes is that sovereign default models tend to display procyclical optimal fiscal policy.\footnote{See Cuadra et al. (2010).} However, fiscal policy is generally countercyclical in developed countries and in Spain in particular.

Second, motivated by high unemployment rates in some European countries, the model features downward nominal wage rigidity as in Schmitt-Grohé and Uribe (2016a). This nominal
rigidity, coupled with a fixed exchange rate, generates a real rigidity that can cause unemployment in equilibrium. Third, it assumes that the government can strategically default on its debt as in Eaton and Gersovitz (1981) and Arellano (2008). To the best of my knowledge, this is the first paper to propose a model with these three characteristics jointly.

I calibrate the model to Spain, one of the countries that implemented important austerity packages after 2010. I then use it to assess the effects of austerity through a counterfactual exercise. I ask what would have happened to GDP, consumption, employment, sovereign spreads and debt to GDP ratios if, instead of implementing fiscal austerity after the second quarter of 2010, Spain had followed historical fiscal rules. I find that there was a significant impact on GDP, consumption and employment. Particularly, relative to the counterfactual, GDP and consumption were 4.5% lower in 2014 as a consequence of austerity. Employment, measured as total hours worked, was 6% lower in 2014. Moreover, the results indicate that fiscal austerity was self-defeating: it did not decrease the debt to GDP ratio (it remained around 90% of GDP in 2014) and actually increased annual sovereign spreads (1 percentage point higher on average after 2010Q2). As intended, austerity reduced the government’s debt level. However, it also decreased GDP in similar proportions leaving the debt to GDP ratio unchanged. This drop in GDP is associated with high fiscal multipliers generated by rigid nominal wages.

The reason why spreads increased is as follows. Since the model assumes that the government can strategically default, the increase in default risk and sovereign spreads is directly related with how austerity impacted the welfare associated with repaying the debt (or the value of repaying) and the welfare of not repaying (or the value of defaulting). The results indicate that austerity reduced the value of repaying even though it decreased debt levels. This is mainly because it worsened the recession and decreased consumption levels making debt repayment less attractive. Moreover, the value of defaulting increased with austerity. This result is mainly related to the assumption that the government is in financial autarky after default. Financial autarky implies that the government is forced to have a balanced budget and therefore the value of government spending equals taxes. Hence, lower government spending as a result of austerity entailed lower taxes causing higher consumption and welfare levels in default. With lower relative welfare from repaying debt, the government’s default probability increased causing the sovereign spread to
Literature Review. This paper is related to three lines of research: (i) fiscal austerity, (ii) models with strategic sovereign default, and (iii) analysis of the Eurozone Crisis.

Regarding fiscal austerity analysis, this paper is most closely related to papers following a structural approach. In that line, House et al. (2015) pursue a similar exercise but without incorporating sovereign debt in their analysis. They find significant output costs as a consequence of austerity. Arellano and Bai (2016) use a sovereign default model to analyze the Greek crisis. However, their model does not display unemployment and they do not use data on fiscal variables to discipline their calibration. Bianchi et al. (2015) is also directly related to this paper. They use a similar model to perform a normative analysis of fiscal policy. They show that wasteful expansionary fiscal policy might be desirable during recessions. However, they do not incorporate sovereign default in their model. Finally, Bi et al. (2013) study the macroeconomic impact of fiscal consolidations when the starting date of an austerity plan is unknown. They find that the composition, monetary policy stance and debt levels are important to determine the final impact.

The fiscal austerity literature has an older empirical branch starting with Giavazzi and Pagano (1990) and Alesina and Ardagna (2010). In these papers the authors use panels of austerity events to identify fiscal austerity shocks. Subsequent developments in this line of research are Alesina et al. (2012) who analyze medium term fiscal plans instead of fiscal policy shocks, and Guajardo et al. (2014) and Alesina et al. (2015) who follow a narrative approach as in Romer and Romer (2010). These papers generally find significant costs in terms of GDP growth, but the size of costs varies depending on the composition of austerity packages. In a similar spirit, Easterly et al. (2008) and Easterly and Servén (2003) remark that fiscal consolidation might have important supply side effects when it is highly focused on public investment. Following a different approach, this paper contributes to this branch of literature by computing the effects of fiscal consolidation on real GDP. Moreover, it also assesses the impact on default risk and sovereign spreads.

The strategic default part of my model is taken from the sovereign default literature. In particular, I model default as in Eaton and Gersovitz (1981), Arellano (2008) and Mendoza...
and Yue (2012). I also adopt the technique for modeling long term debt from Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Hatchondo et al. (2015) incorporate fiscal rules into a sovereign default model, but in the form of debt to GDP and sovereign spreads ceilings. They use their model to perform a different exercise. They assess the impact of debt and spreads ceilings on equilibrium sovereign spreads.

There are several papers that analyze the recent European crisis. This paper contributes to this literature by analyzing how fiscal policy after 2010 affected the severity of the crisis. Lane (2012) and Shambaugh (2012) provide a detailed description. Martin and Philippon (2014) use a structural DSGE model to assess the different factors that might have caused the crisis. They find that pre-crisis fiscal policy was an important ingredient. Like this paper, Schmitt-Grohé and Uribe (2016a) highlight the importance of downward nominal wage rigidity as a propagation factor in Europe.

Outline. The rest of the paper is organized as follows. Section 2 to 5 describe the model, section 6 highlights the key mechanisms, section 7 describes the calibration and 8 shows the results. Section 9 concludes.

2 Model

The model is in discrete time and describes an economy populated with four different agents: Households, Firms, International Creditors and a Government.

2.1 Household

There is a representative Household that chooses consumption ($C_t$) and labor ($H_t$) plans to maximize a time separable utility,

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\frac{1}{\theta}}}{1 + \frac{1}{\theta}} \right\}$$

(1)
where $\sigma$ is the risk aversion coefficient, $\chi$ is a labor disutility parameter and $\theta$ denotes the Frisch elasticity. Consumption ($C_t$) is a composite of nontradable and tradable goods defined by the following Armington aggregator with elasticity of substitution $\mu$ and nontradables weight $\omega$, 

$$C_t = C(C_{Nt}, C_{Tt}) = \left[ \omega (C_{Nt})^{\mu-1 \mu} + (1 - \omega) (C_{Tt})^{\mu-1 \mu} \right]^{\mu-1 \mu}$$  

(2) 

$C_{Nt}$ and $C_{Tt}$ denote nontradable and tradable consumption, respectively. The budget constraint that this agent faces is given by,

$$P_C C_t = W_t H_t + P_T T_t$$  

Where $W_t$ is the nominal wage, $P_T$ and $P_{Nt}$ represent tradable and nontradable prices. Further, $\Pi_t$ are firm profits and $T_t$ denotes lump sum taxes, both expressed in units of tradable goods. $P_C$ is the consumer price level and is defined by,

$$P_C = \left[ \omega^\mu P_{Nt}^{1-\mu} + (1 - \omega)^\mu P_{Tt}^{1-\mu} \right]^{\frac{1}{1-\mu}}$$ 

The tradable good has a constant price $P_{Tt} = P^*$ and is the numeraire in this economy. I use lower cases to denote relative prices with respect to $P_{Tt}$, i.e. $p_{Nt} = \frac{P_{Nt}}{P_{Tt}}$, $p_{Ct} = \frac{P_{Ct}}{P_{Tt}}$ and $w_t = \frac{W_t}{P_{Tt}}$. Therefore, the budget constraint and consumer price definition can be re-expressed as,

$$p_{Nt} C_{Nt} + C_{Tt} = w_t H_t + \Pi_t - T_t$$  

(3) 

$$p_{Ct} = \left[ \omega^\mu p_{Nt}^{1-\mu} + (1 - \omega)^\mu \right]^{\frac{1}{1-\mu}}$$  

(4) 

### 2.2 Firms

There are two firms that employ labor to produce nontradable and tradable goods. They use the following production technologies,
\[ Y_{Nt} = A_{Nt}(H_{Nt}^d)^{\alpha_N} \]  
\[ Y_{Tt} = A_{Tt}(H_{Tt}^d)^{\alpha_T} \]

\( Y_{Nt} \) and \( Y_{Tt} \) are the production levels in every sector. Productivity levels are denoted by \( A_{Nt} \) and \( A_{Tt} \), and labor demand in each sector is represented by \( H_{Nt}^d \) and \( H_{Tt}^d \). I assume that productivity follow AR(1) processes,

\[ \log(A_{Nt}) = \rho_N \log(A_{Nt-1}) + \sigma_N \epsilon_{Nt}^A \]  
\[ \log(A_{Tt}) = \rho_T \log(A_{Tt-1}) + \sigma_T \epsilon_{Tt}^A \]

Firms hire labor and maximize the following profit functions,

\[ \Pi_{Tt} = A_{Tt}(H_{Tt}^d)^{\alpha_T} - w_t H_{Tt}^d \]  
\[ \Pi_{Nt} = p_{Nt} A_{Nt}(H_{Nt}^d)^{\alpha_N} - w_t H_{Nt}^d \]

2.3 Labor Market

As in Schmitt-Grohé and Uribe (2016a), I introduce downward nominal wage rigidity in the labor market. In particular, I assume that nominal wages at time \( t \) cannot be lower than a proportion, \( \gamma \), of the wage level at \( t - 1 \).

\[ w_t \geq \gamma w_{t-1} \]  

Constraint (11) implies that this model can display unemployment whenever the constraint binds.
2.4 Government

I assume the government is benevolent and has different policy instruments: government consumption \( G_t \), lump sum net taxes \( T_t \), and the decision of defaulting on its debt \( d_t \).

For simplicity, I assume government consumption is allocated to tradables and nontradables in fixed proportions. In other words, I assume \( G_t \) is given by,

\[
G_t = \min \left\{ \frac{G_{Nt}}{\phi_g}, \frac{G_{Tt}}{1 - \phi_g} \right\}
\]  

(12)

where \( \phi_g \) is the percentage of \( G_t \) that is allocated to nontradables. As a result, the government consumption price (relative to tradable goods) is given by,

\[
p_{Gt} = \phi_g p_{Nt} + (1 - \phi_g)
\]  

(13)

Moreover, the government issues long term debt to finance deficits. As in Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), the government can issue bonds with a geometrically decaying coupon \( \delta \). In particular, a bond issued at time \( t \) promises a stream of coupon payments \( \delta(1 - \delta)^{i-1} \) in periods \( t + i \) for \( i \geq 1 \). Hence, government budget constraint is given by,

\[
p_{Gt}G_t + \delta B_t = T_t + q_t [B_{t+1} - (1 - \delta)B_t]
\]  

(14)

where \( B_t \) is the face value of debt, \( q_t \) is the price of debt that international creditors are willing to pay, \( \delta B_t \) represent the coupon payments the government needs to pay at time \( t \), and \( B_{t+1} - (1 - \delta)B_t \) is the amount of debt units issued (if \( B_{t+1} \geq (1 - \delta)B_t \)) or purchased (if \( B_{t+1} < (1 - \delta)B_t \)).

2.5 International Creditors

As it is usual in the sovereign default literature, international creditors are deep pocket. They have a stochastic discount factor \( M_t = \tilde{\beta}m_t \), where \( m_t \) is a risk aversion shock. No arbitrage implies the following pricing equation,
\[ q_t = \tilde{\beta} E_t \left\{ m_{t+1} \left\{ (1 - d_{t+1}) \left( \delta + (1 - \delta) q_{t+1} \right) + d_{t+1} (1 - \psi) \right\} \right\} \]  

(15)

As expressed in equation (15), \( q_t \) is the discounted expected payoff flow at \( t + 1 \). Future payments depend on the default decision \( d_{t+1} \). Hence, if government does not default \( (d_{t+1} = 0) \) creditors receive a coupon payment \( \delta \) and the market value of outstanding debt \( (1 - \delta) q_{t+1} \). On the other hand, if government defaults creditors receive the face value of debt with an exogenous haircut rate \( \psi \).\(^4\)

The risk aversion shock \( m_t \) is assumed to follow a AR(1) process in logs,

\[ \log(m_t) = \rho_m \log(m_{t-1}) + \sigma_m \epsilon_t^m \]

where \( \epsilon_t^m \) is a standard normally distributed shock.

### 3 Implementable Equilibrium

Given \( B_0, A_{N0}, A_{T0}, m_0, G_0 \), a sequence of fiscal policy variables \( \{G_t, T_t, B_{t+1}, d_t\}_{t=0}^\infty \) and a shocks sequence \( \{A_{N,t+1}, A_{T,t+1}, m_{t+1}\}_{t=0}^\infty \), an equilibrium is a set of prices \( \{w_t, p_{Nt}, p_{Ct}, q_t\}_{t=0}^\infty \) and allocations \( \{H_{Nt}, H_{Tt}, H_t^s, C_{Nt}, C_{Tt}\} \) such that,

1. Household maximizes utility (1) subject to (3)

2. Firms maximize profits (9) and (10)

\(^4\)This assumption is made for simplicity but it is common practice in quantitative sovereign default models. For models with endogenous haircuts see: Benjamin and Wright (2013) and Yue (2010)
3. Markets clear

\[ G_{Nt} + C_{Nt} = Y_{Nt} \]  
\[ C_{Tt} + G_{Nt} + \delta B_t = Y_{Tt} + q_t [B_{t+1} - (1 - \delta)B_t] \]  
\[ (w_t - \gamma w_{t-1})(H^s_t - H^d_{Nt} - H^d_{Tt}) = 0 \]  
\[ w_t \geq \gamma w_{t-1} \]

4. \( q_t \) satisfies (15)

Note that market clearing in labor markets (equations (18) and (19)) is not standard because of downward nominal wage rigidity. In particular, labor demand \( H^d_{Nt} + H^d_{Tt} \) determines the equilibrium total working hours. If the nominal wage rigidity constraint is slack \( w_t > \gamma w_{t-1} \) labor demand equals supply \( (H^s_t) \) and there is no unemployment. In turn, if the constraint binds total hours equal \( H^d_{Nt} + H^d_{Tt} \) and there is positive unemployment as \( H^s_t > H^d_{Nt} + H^d_{Tt} \).

4. Government Policy

The government has access to three different instruments: a sovereign default decision \( d_t \) that affects the price of debt \( q_t \), government consumption \( G_t \), and net taxes \( T_t \). The interaction of these three instruments determines the evolution of the face value of debt \( B_t \) through the government budget constraint.

I assume that the government follows fiscal rules to determine \( G_t \) and \( T_t \). These fiscal rules describe fiscal policy in normal times and are calibrated using fiscal data before the Great Recession.

The rules determine a primary deficit \( p_G G_t - T_t \). This deficit together with debt repayments are the financial needs that the government need to cover by issuing new debt. The government might decide to default on its debt because of two different reasons. First, it might default because financial needs are too high and as result the government is not able to get sufficient credit from the markets. This happens when the financial needs are higher than the peak of the debt Laffer Curve, that is when,
\[ p_{Gt}G_t - T_t + \delta B_t > \max_{B_{t+1}} \{ q_t(B_{t+1}) [B_{t+1} - (1 - \delta)B_t] \} \]

where the right hand side of (20) represents the maximum amount the government could get from international markets (the peak of the debt Laffer curve).

I also assume that the government can strategically choose to default even when the government is able to get the necessary funds to repay the debt. This default decision is modeled as in typical sovereign default models à la Eaton and Gersovitz (1981). Hence, every period the government decides whether to repay the debt and keep having access to international debt markets, or to default and stay in autarky for a random number of periods bearing productivity costs.

4.1 Fiscal Rules

Following Blanchard and Perotti (2002), I assume that government consumption \( G_t \) evolves according to the following fiscal rule,

\[ \log(G_t) = (1 - \rho_G)\log(\bar{G}) + \rho_G\log(G_{t-1}) + \rho_{GY} [\log(Y_{t-1}) - \log(\bar{Y})] + \sigma_G \epsilon_t^G \]

where \( \epsilon_t^G \) is a standard normally distributed fiscal shock, \( \bar{G} \) represents the steady state government consumption, \( Y_{t-1} \) is lagged output and \( \bar{Y} \) is steady state output. This fiscal rule includes lagged GDP in order to allow for the possibility of having counter-cyclical government consumption. Hence, the government might find desirable to implement expansionary fiscal policies during a recession. The implicit assumption in this rule is that the government can only react to changes in GDP with a lag of one quarter.

Moreover, following Bi (2012), net taxes are defined by the following fiscal rule with parameters \( t^* \) and \( \gamma_T \)

\[ \frac{T_t}{Y_t} = t^* + \gamma_T \left( \frac{B_t}{4Y_t} \right) \]

where \( Y_t \) is GDP and \( B_t \) is the stock of sovereign debt, both variables are expressed in units of
tradable goods.\textsuperscript{5} As shown in figure 1, there is positive and significant relationship between net taxes and debt levels in most of Eurozone countries with a sufficiently long series of Debt to GDP ratios. The exception is Netherlands that does not show a statistically significant relationship.

The rule seems intuitive as higher debt levels imply higher debt repayments and, hence, more financial needs. According to the rule, the government decides to rise taxes to face those needs.

### 4.2 Default Decision

Conditional on having repayment capacity, the Government decides whether to repay or default in order to maximize Household’s welfare. This welfare maximization is constrained by the implementable equilibrium conditions defined in section 3 and the fiscal rules defined in section 4.1. As it is usual in the sovereign default literature I focus on a Markov Perfect Equilibrium. This equilibrium definition implies that the default decision depends on a set of states $S$, i.e. $d = \Phi(S)$. Moreover, the government today decides to repay or default taking as given the default decision tomorrow $d' = \Phi(S')$.

The relevant set of states is given by $S \equiv \{S_{1}, S_{2}\}$, where $S_{1} \equiv \{B, w_{-1}\}$ and $S_{2} \equiv \{A_{N}, A_{T}, G, m\}$. $S_{1}$ is composed by the two endogenous states in the economy, the face value of debt and the lagged wage level. Further, $S_{2}$ groups the exogenous variables that hit the economy and follow stochastic processes described in section 2.

Let $V(S)$ be the government’s value function before taking the repayment/default decision. Further, let $V^{R}(S)$ and $V^{D}(S)$ be the value functions after deciding to repay and default, respectively. Define $C(S), H(S), Y(S), p_{G}(S), w(S)$ as consumption, hours, total product, price of government consumption and wage rate consistent with the implementable equilibrium. Hence, for a given price function $q(S'_{1}, S_{2})$ where $S'_{1} \equiv \{B', w'_{-1}\}$ the value functions satisfy,

\textsuperscript{5}Bi (2012) finds a positive relationship between debt levels and taxes. I find that this relationship still applies when using net taxes (Tax Income minus Transfers)
\[ V(S) = \max_{d \in \{0,1\}} \left\{ (1 - d)V^R(S) + dV^D\left( (1 - \psi)B, w_{-1}, A_N, A_T, (1 - d_g)G, m \right) \right\} \] (23)

subject to

\[ d = 1 \text{ if } p_G(S)G(S) - T(S) + \delta B > \max_{B'} \left\{ q(S_1', S_2) \left[ B' - (1 - \delta)B \right] \right\} \]

\(V(S)\) summarizes the default/repayment decision. Every period the government compares the value of repaying \(V^R\) with the value of defaulting \(V^D\) and acts accordingly. Notice that the value of defaulting has two different parameters (\(\psi\) and \(d_g\)) modifying its arguments. These two parameters highlight two different things that happen at the default event. First, \(\psi\) is an exogenous haircut to the face value of debt as a result of the default. Second, there is a drop \((d_g)\) in government consumption, where \(d_g\) is calibrated to match the average drop in \(G\) in historic default episodes. These changes in \(B\) and \(G\) are one time events and only happen at the default episode. The constraint in (23) highlights the fact that the government decides whether to default or not only when it has repayment capacity. If that is not the case, the government automatically defaults.

The value of repayment \(V^R(S)\) satisfies,

\[ V^R(S) = u(C(S), H(S)) + \beta E \{ V(S')|S \} \] (24)

subject to

\[ p_G(S)G + \delta B = T(S) + q(S_1', S_2) \left[ B' - (1 - \delta)B \right] \]

\[ T(S) = t'Y(S) + \frac{\gamma t}{4}B \]

\[ w_{-1}' = w(S) \]

\[ \log(G') = (1 - \rho_G)\log(\bar{G}) + \rho_G \log(G) + \rho_{GY} \left[ \log(Y(S)) - \log(\bar{Y}) \right] + \sigma_G \epsilon^G \]

+ Equilibrium Conditions

The value of repayment where consists of an instantaneous utility \(u(C(S), H(S))\) and a
discounted continuation value \( E \{ \mathcal{V}(S')|S \} \). The first constraint in (24) is the government budget constraint and the second describes the net tax rule. These two conditions determine the law of motion for the debt face value \( B \). The third constraint describes the law of motion for lagged wages \( w_{-1} \), it simply states that future lagged wages are equal to wages today. The forth constraint shows the fiscal rule for government spending \( G \), and the fifth line highlights the fact that private allocations are consistent with the implementable equilibrium.

The value in default \( \mathcal{V}^D(S) \) is given by,

\[
\mathcal{V}^D(S) = u \left( C(S_1, \tilde{S}_2), H(S_1, \tilde{S}_2) \right) + \beta E \left\{ \phi \mathcal{V}(S') + (1 - \phi) \mathcal{V}^D(S')|S \right\}
\]

st.

\[
T(S_1, \tilde{S}_2) = p_G(S_1, \tilde{S}_2)G
\]

\[
B' = BR^* 
\]

\[
w'_{-1} = w(S_1, \tilde{S}_2)
\]

\[
\log(G') = (1 - \rho_G)\log(\bar{G}) + \rho_G\log(G) + \rho_{GY} [\log(Y(S)) - \log(\bar{Y})] + \sigma_G e^G
\]

+ Equilibrium Conditions

\( \mathcal{V}^D(S) \) consists of an instantaneous utility level plus a continuation value. In this case, the continuation value includes an exogenous probability of coming back to international capital markets, \( \phi \). The first constraint in (25) is the government budget constraint, where the value of government spending equals net taxes. The second highlights the assumption that, once in default, the stock of debt that the government owes is updated by the international risk free rate \( R^* = 1/\bar{\beta} \). The third constraint is the law of motion for wages, and the forth represents the government spending fiscal rule. Hence, this model assumes that once in default, the government follows the government spending fiscal rules and adjusts net taxes in order to keep a balanced budget.

Moreover, \( \tilde{S}_2 \equiv \{ \tilde{A}_N, \tilde{A}_T, G, m \} \) in (24) is a modified state vector that includes exogenous productivity costs of default.\(^6\) In particular, I follow Chatterjee and Eyigungor (2012) and

\(^6\)This is a typical assumption in sovereign default models. Mendoza and Yue (2012) provide a microfoundation
assume the following productivity levels in default,

\[
\tilde{A}_N = A_N - \max \left\{ 0, d_0 A_N + d_1 A_N^2 \right\}
\]

\[
\tilde{A}_T = A_T - \max \left\{ 0, d_0 A_T + d_1 A_T^2 \right\}
\]

As it is usual in the sovereign default literature, productivity costs are convex. This is key assumption to generate incentives to default when the economy is facing low productivity levels, and therefore low GDP. Since the productivity costs of default are convex, lower productivity level imply a lower cost of default.

The government has also more incentives to default when sovereign debt is high. High debt generates high net taxes through the tax rule and, therefore, low consumption. In turn, low consumption means that there is a high marginal utility for consuming, which implies potentially high utility gains from defaulting and reducing taxes.

The nominal wage level affects the default decision as well. Particularly, lower nominal wages in general imply lower costs of default and therefore, more incentives to decide not to repay. There is a drop in productivity levels when the country defaults. This drop reduces the demand for labor generating unemployment. The lower wages are, the less important the increase in unemployment is. Hence, a lower unemployment level makes defaulting more attractive.

A higher risk premium is related with higher default probabilities as it increases the cost of issuing debt. Finally, the effect of government spending on spreads is ambiguous a priori. I will come back to the effects of austerity on spreads in section 8, but bear in mind that a drop in $G$ has opposite effects on spreads because it reduces debt levels on one hand, but on the other it might affect private equilibrium allocations in a negative way, reducing consumption levels and nominal wages.

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for this assumption. They claim that during financial autarky firms are not able to import certain intermediate goods, generating lower levels of productivity.
5 Recursive Equilibrium

The recursive Markov Perfect Equilibrium incorporates the optimal default decision of the government subject to the implementable equilibrium defined in section 3.

A Markov Perfect Equilibrium is a set of value functions $V(S), V^R(S), V^D(S)$, a default decision $d = \Phi(S)$ and a price schedule $q(S'_1, S_2)$ such that,

1. Given the price schedule $q(S'_1, S_2)$, the value functions solve problems (23), (24) and (25)

2. Given the default decision, the price schedule satisfy the following equation

$$ q(S'_1, S_2) = \beta E \{ m' \left( (1 - \Phi(S')) (\delta + (1 - \delta)q(S'_1, S'_2) ) + \Phi(S')(1 - \psi) \right) | S'_1, S_2 \} \quad (26) $$

3. The default decision solves problem (23)

As noted by Aguiar and Amador (2016) there might be multiple equilibria in default models with long term debt. I analyze the equilibrium that arises as a limit of a finite horizon economy to deal with this potential problem.

6 Analysis

The model described in the previous sections is rich enough to display non linear responses to fiscal policy. As a consequence, the effect of changes in government expenditure depends on what part of the state space the economy is located. In this section I describe the mechanisms behind fiscal policy impact. In particular, I explain the main mechanism that determines the size and sign of fiscal multipliers.

What is the impact of a decrease in government consumption in this model? To provide some intuition consider the following equation that comes from Household first order conditions with respect to consumption of nontradables $C_{Nt}$ and tradables $C_{Tt}$,

---

7 Auclert and Rognlie (2016) show uniqueness only for particular cases of sovereign default à la Eaton and Gersovitz (1981) and Arellano (2008) models with short term debt.
8 Appendix A.3 describes the numerical routine used to solve the model.
\[ p_{Nt} = \frac{\omega}{1 - \omega} \left( \frac{C_{Tt}}{C_{Nt}} \right)^{1/\mu} \]
\[ \downarrow p_{Nt} = \frac{\omega}{1 - \omega} \left( \frac{C_{Tt}}{Y_{Nt} - \downarrow G_{Nt}} \right)^{1/\mu} \] (27)

where the second line in (27) plugs in the market clearing in nontradable goods market. Equation (27) implies that, for a given level of \( C_{Tt} \) and nontradables output \( Y_{Nt} \), the impact of a drop in government spending on nontradables \( G_{Nt} \) is a reduction in the relative price of nontradables \( p_{Nt} \). The drop in \( p_{Nt} \) has an effect on the labor market. In particular, consider the labor demand for tradables and nontradables that come from maximizing (9) and (10), respectively.

\[ H_{Tt}^{d} = \left( \frac{\alpha T A_{Tt}}{w_{t}} \right)^{\frac{1}{1-\alpha T}} \] (28)
\[ \downarrow H_{Nt}^{d} = \left( \frac{\downarrow p_{Nt} \alpha N A_{Nt}}{w_{t}} \right)^{\frac{1}{1-\alpha N}} \] (29)

Equation (29) shows that a drop in \( p_{N} \) generates a fall in the nontradable sector labor demand. This fall in labor demand generates a drop in total GDP. However, the magnitude of the final impact on GDP will depend on the downward nominal wage rigidity constraint. If the constraint does not bind the total effect is mitigated by a drop in wages. Lower wages increase labor demand in the tradable sector and reduce the initial drop in \( H_{Nt} \). However, this second effect is not present when the constraint binds. Hence, since wages fall little, production in the tradable sector is marginally affected and GDP drops more as a result.

---

9Here I focus on a change in \( G_{N} \) instead of \( G \) because, as I will detail in the calibration section, most of government consumption is spent on nontradables.
The two different cases are depicted in Figure 2. Panel (a) shows the case in which wages fall reducing the total impact on quantities. On the other hand, panel (b) illustrates the case in which the downward nominal wage constraint binds and wages can fall only up to $\gamma w_0$. In this second case the impact of the fiscal shock is much more important. As wages can not adjust, labor demand ends up being lower than supply resulting in positive unemployment levels ($H^*_s > H^*_r$).

The previous analysis shows that fiscal multipliers can be highly non linear and state dependent. But how big can they be with reasonable parameter values? Table 1 shows the impact multipliers for different changes in government consumption (in direction and magnitude). The Impact multiplier is defined as,

$$\mu_G = \frac{\Delta GDP_t}{\Delta G_t} = \frac{\bar{p}_N \Delta N_t + \Delta Y_t}{\Delta G_t}$$

where $\bar{p}_N$ is the steady state relative price of nontradable to tradable goods, $\Delta GDP_t$ and $\Delta G_t$ represent the change in real GDP and government spending. The columns in Table 1 compute $\mu_G$ for $\Delta G$’s of different magnitudes and sign, assuming an initial point in which the constraint does not bind. The rows compute the multiplier at different points of the state space. Particularly, I analyze the change in multipliers for different amounts of debt and productivity.
levels in both sectors. In the case of productivity in the tradable and nontradable sectors, "High" and "Low" represent levels four standard deviations above and below the steady state value. When I change debt levels, "High" and "Low" denote debt levels 20% above and below the steady state.

Table 1: Impact Multipliers and Fundamentals

<table>
<thead>
<tr>
<th>States</th>
<th>↓ 5% $G$</th>
<th>↓ 1% $G$</th>
<th>↓ 0.1% $G$</th>
<th>↑ 1% $G$</th>
<th>↑ 5% $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State</td>
<td>2.81</td>
<td>1.91</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.08</td>
</tr>
<tr>
<td>Nontradables productivity $A_N$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2.68</td>
<td>1.57</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Low</td>
<td>2.94</td>
<td>2.26</td>
<td>-0.19</td>
<td>-0.20</td>
<td>-0.22</td>
</tr>
<tr>
<td>Tradables productivity $A_T$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2.84</td>
<td>1.97</td>
<td>-0.15</td>
<td>-0.16</td>
<td>-0.17</td>
</tr>
<tr>
<td>Low</td>
<td>2.77</td>
<td>1.85</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Debt level $B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2.78</td>
<td>1.79</td>
<td>-0.05</td>
<td>-0.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>Low</td>
<td>2.83</td>
<td>2.03</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

Notes. The table shows impact multipliers when government spending changes by different amounts (as percentage of steady state government spending) and directions. These different cases are shown in different columns. The rows imply different parts of the state space. The first row shows multipliers at the steady state. The rest of the columns compute the multipliers for different levels of productivity and debt levels. High and low productivity levels (in tradable and nontradable sectors) means four standard deviations above and below the mean. High and low debt cases correspond to debt levels 20% above and below the steady state, respectively.

Table 1 shows that the multipliers are close to zero when government spending increases or decreases by a small amount so that the constraint does not bind (columns 3 to 6). In these cases government spending affects GDP through two channels: wealth effects and reallocation of labor across sectors. Wealth effects affect GDP through changes in labor supply: a higher $G$ brings about higher taxes $T$ which in turn reduce demand for leisure, shifting labor supply rightwards. In addition, since most of $G$ is devoted to nontradable goods, a higher $G$ increases relative demand for nontradable goods shifting labor to this sector. The total effect of this reallocation
on GDP depends on the relative labor productivity of each sector. If the nontradables labor productivity is higher than that of tradables, this shift increases GDP and vice versa. In general, Table 1 shows that multipliers are negative when the constraint does not bind. This means that a drop in government spending can increase GDP. However, they can become positive when the productivity level in the nontradable sector is considerably higher than that of tradables (see rows 2 and 5).

As presented in Table 1, multipliers are much higher when the constraint binds. Hence, output contracts more when there is a drop in $G$. They are between 1.5 and 3 when the reduction in $G$ is large enough (column 1 and 2). This values are higher than the ones found in the empirical literature on average. However, they are in line with fiscal multipliers in recessions that are found using regime switching VARs.\footnote{See for example Auerbach and Gorodnichenko (2012), Nicoletta Batini and Melina (2012) and Owyang et al. (2013).} Note that the value of multipliers when the constraint binds changes for different points in the state space. This is due to the fact that the initial impact of $G$ on $p_N$ varies across the state space. From equation (27) one can see that the effect of $G$ on $p_N$ is higher when tradables consumption $C_T$ is relatively high and nontradables output $Y_N$ low. This explains why the multiplier is higher when tradables productivity $A_T$ is high and nontradables productivity $A_N$ is low. Moreover, a high level of debt is associated with high debt repayments, high exports of tradable goods and lower $C_T$, implying a lower multiplier.

7 Calibration

I calibrate the model using data from Spain, one of the countries with most important austerity programs in Europe. Standard parameters are set using common values in the literature. The rest of the parameters are chosen to match a set of moments for Spain.

Risk aversion $\sigma$, the Frisch elasticity $\theta$, and output labor elasticities $\alpha_N$ and $\alpha_T$ are calibrated to standard values. Household’s discount factor $\beta$ is 0.93, a common value in the default literature. The labor disutility parameter $\chi$ is set to get a steady state labor equal to one. In addition, the elasticity of substitution between tradables and nontradables $\mu$ comes from Stockman and Tesar (1995) and corresponds to a sample of industrialized countries. The weight of
nontradables is used to match the share of nontradables value added on total GDP in steady state. The downward nominal wage rigidity parameter $\gamma$ is set within the range of values for Europe shown in Schmitt-Grohé and Uribe (2016a). This parameter implies a maximum annual drop in wages of 1%, which is reasonable given the fact that Spanish nominal wages decreased by 0.5% annually on average from 2010 to 2014.

Regarding the government sector, fiscal rules are estimated using data before the financial crisis. In particular, I use the data from 1970 to 2007 for the fiscal rule on government spending $G$ and data from 1980 to 2007 for the tax rule. The estimated coefficient that relates $G$ with output $Y$ in (21) is economically insignificant. For that reason, I assume that $G_t$ follows a simple AR(1) process when I solve the model. The proportion of government expenditure that is devoted to nontradables $\phi_g$ is difficult to calibrate given the lack of detailed data. I assume that $\phi_g$ equals 0.9 using the fact that 60% of $G$ is public employment, which is nontradable. I then divide the remaining 40% into tradables and nontradables using the private nontradable consumption weight of 0.81 ($\omega$). I calibrate the drop in $G$ in a default event ($d_g$) to match the reduction in government spending in historical defaults episodes. This episodes are taken from Schmitt-Grohé and Uribe (2016b) and correspond to developing countries mostly.

The coupon rate is set to match a 6 year average debt maturity. International creditor’s discount factor is such that the annual risk free rate is 4%. The default haircut $\psi$ is taken from Cruces and Trebesch (2013) and refers to the average haircut after debt restructurings involving face value reductions (65%). The probability of returning to markets $\phi$ matches the average exclusion time computed by Cruces and Trebesch (2013). I am using the average exclusion time related to restructurings with haircuts higher than 30%.

I estimate the productivity costs of default parameters $d_0$ and $d_1$ and the shock processes by fitting the moments listed in table 3. The calibration aims to match average spreads, average and standard deviation of debt to GDP ratio. In addition I run three simple regressions using data from Spain on spreads ($s_t$), nontradables ($Y_N$) and tradables output ($Y_T$). The estimated

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11 Data from Instituto Nacional de Estadísticas (INAE) for the period 1997Q1-2014Q2. The tradable sector refers to Agriculture and Industry, whereas the nontradable part of the economy includes Construction and Services.

12 Detrended nominal wages. Data from the Spanish Statistics Agency (INE).

13 It is around 0.02, whereas the AR(1) coefficient in the rule is 0.98.

14 Data from the Spanish Treasury, see http://www.tesoro.es/sites/default/files/estadisticas/02I.pdf
equations are (31), (32) and (33), where $\bar{s}$, $\bar{Y}_N$ and $\bar{Y}_T$ are sample means. I use the AR(1) coefficients and standard deviation of residuals coming from these regressions as target moments.

\begin{align*}
  s_t - \bar{s} &= \rho_s (s_{t-1} - \bar{s}) + \mu_{st} \\
  \log(Y_{Nt}) - \log(\bar{Y}_N) &= \rho_N (\log(Y_{Nt-1}) - \log(\bar{Y}_N)) + \mu_{Nt} \\
  \log(Y_{Tt}) - \log(\bar{Y}_T) &= \rho_T (\log(Y_{Tt-1}) - \log(\bar{Y}_T)) + \mu_{Tt}
\end{align*}

(31)  
(32)  
(33)

In order to get the model implied estimates of regressions (31), (32) and (33), I simulate the model 10,000 times and estimate the equations for windows of time in which the government is not in default.\(^\text{15}\) The moments in table 3 are the median values of all windows. The distance between the data and model implied moments is weighted by the data Bootstrap standard errors of each moment. Therefore, moments that are not precisely estimated with real data receive a lower weight in the calibration.

**Model Fit.** The model does a decent job fitting the selected moments, especially in terms of average spreads, debt levels and serial correlation coefficients. The model implied standard deviation of spread residuals is higher than the ones estimated using real data. This is an expected outcome given the fact that from 1995 to 2007 Spanish spreads were steadily close to zero with very small changes around the mean.

Moreover, the standard deviations of residuals from the nontradables and tradables output regressions (32) and (33) are higher than the data implied ones. This difference is mainly because the model does not assume any friction in the reallocation of labor across sectors. More reallocation of labor across sectors implies more volatility in total value added in each sector.

Following Bocola and Dovis (2016), I check the model fit by comparing the relationship between fundamentals and spreads. Figure 3 shows the model implied relationships and the data points for Spain in the period 1995-2014. The black solid line is a fitted fourth degree

\(^{15}\)I only consider windows with more than 30 observations. Also, I exclude from the analysis the first 30 observations after a default event.
Table 2: Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Household, Firms and Labor Market</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.93</td>
<td>Standard</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\sigma$</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>Frisch Elasticity</td>
<td>$\theta$</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>Labor disutility</td>
<td>$\chi$</td>
<td>1.52</td>
<td>Steady State Labor $\bar{H} = 1$</td>
</tr>
<tr>
<td>Elast. of substitution</td>
<td>$\mu$</td>
<td>0.74</td>
<td>Stockman and Tesar (1995)</td>
</tr>
<tr>
<td>Nontradables weight</td>
<td>$\omega$</td>
<td>0.81</td>
<td>SS Nontradables Output Share = 88%</td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>$\alpha_{N,\alpha_{T}}$</td>
<td>2/3</td>
<td>Standard</td>
</tr>
<tr>
<td>Downward wage rigidity</td>
<td>$\gamma$</td>
<td>0.99^{1/4}</td>
<td>Schmitt-Grohé and Uribe (2016a)</td>
</tr>
<tr>
<td><strong>Panel B: Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady State G</td>
<td>$\bar{G}$</td>
<td>0.22</td>
<td>Steady State G/Y = 19%</td>
</tr>
<tr>
<td>G rule AR(1)</td>
<td>$\rho_G$</td>
<td>0.986</td>
<td>Fiscal Rule Estimation</td>
</tr>
<tr>
<td>G rule SD</td>
<td>$\sigma_G$</td>
<td>0.93%</td>
<td>Fiscal Rule Estimation</td>
</tr>
<tr>
<td>Net Tax rule parameter</td>
<td>$t^*$</td>
<td>0.18</td>
<td>Fiscal Rule Estimation</td>
</tr>
<tr>
<td>Net Tax rule parameter</td>
<td>$\gamma_T$</td>
<td>0.06</td>
<td>Fiscal Rule Estimation</td>
</tr>
<tr>
<td>Bond coupon rate</td>
<td>$\delta$</td>
<td>1/24</td>
<td>Avg maturity = 6 yrs</td>
</tr>
<tr>
<td>Nontradables share G</td>
<td>$\phi_g$</td>
<td>0.9</td>
<td>Public employment + other nontradables share</td>
</tr>
<tr>
<td>Drop of G in default</td>
<td>$d_g$</td>
<td>0.95</td>
<td>Figure 4(a)</td>
</tr>
<tr>
<td><strong>Panel C: Int’l Creditors and Default</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Int creditors Disc Factor</td>
<td>$\bar{\beta}$</td>
<td>0.99</td>
<td>Annual risk free rate = 4%</td>
</tr>
<tr>
<td>Default Haircut</td>
<td>$\psi$</td>
<td>0.65</td>
<td>Cruces and Trebesch (2013)</td>
</tr>
<tr>
<td>Prob. of reentry</td>
<td>$\phi$</td>
<td>0.04</td>
<td>Cruces and Trebesch (2013)</td>
</tr>
<tr>
<td>Default Prod Cost</td>
<td>$d_0$</td>
<td>-0.57</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Default Prod Cost</td>
<td>$d_1$</td>
<td>0.64</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td><strong>Panel D: Shock Processes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tradables Prod AR(1)</td>
<td>$\rho_{AT}$</td>
<td>0.97</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Nontradables Prod AR(1)</td>
<td>$\rho_{AN}$</td>
<td>0.97</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Risk Aversion AR(1)</td>
<td>$\rho_m$</td>
<td>0.56</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Tradables Prod SD</td>
<td>$\sigma_{AT}$</td>
<td>1.68%</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Nontradables Prod SD</td>
<td>$\sigma_{AN}$</td>
<td>1.83%</td>
<td>Moments in table 3</td>
</tr>
<tr>
<td>Risk Aversion SD</td>
<td>$\sigma_m$</td>
<td>2.75%</td>
<td>Moments in table 3</td>
</tr>
</tbody>
</table>
polynomial using model simulated data. The blue data points correspond to data before 2012, whereas the red ones refer to quarters after that year. Figure 3 shows a good fit with the data if we only consider data points before 2012. We can see there is a strong nonlinear relationship between GDP, debt to GDP ratios and spreads (see panels a and b) if we focus on the blue dots. This model implied regression cannot fit the data points after 2012 simply because spreads dropped even though GDP had the lowest values in the sample and debt to GDP ratios were the highest. The reason of this drop in spreads after 2012 is related to the European Central Bank implementation of Outright Monetary Transactions (OMT) program. This program was launched by the European Central Bank (ECB) in 2012, and basically allowed the ECB to buy sovereign bonds in order to keep sovereign spreads low. The program has not been used so far, but its announcement might have had important effects on international creditors risk aversion. This is incorporated in my model as an external factor affecting sovereign spreads and is captured by the risk premium shock.

I am also including a third panel (c) to check the relationship between government spending and spreads. As will be clear in the results section, the relationship between $G$ and spreads is strongly state dependent and can be positive or negative depending on the effects of $G$ on the

<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
<th>Data Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Spread</td>
<td>0.87%</td>
<td>1.18%</td>
<td>10 yr bond spread 1995Q1-2014Q2</td>
</tr>
<tr>
<td>AR(1) coefficient spread</td>
<td>0.90</td>
<td>0.94</td>
<td>10 yr bond spread 1995Q1-2014Q2</td>
</tr>
<tr>
<td>SD spread</td>
<td>1.04%</td>
<td>0.32%</td>
<td>10 yr bond spread 1995Q1-2014Q2</td>
</tr>
<tr>
<td>Average Debt/GDP</td>
<td>70%</td>
<td>70%</td>
<td>Debt to GDP 1995-2015</td>
</tr>
<tr>
<td>Std Dev Debt/GDP</td>
<td>24%</td>
<td>22%</td>
<td>Debt to GDP 1995-2015</td>
</tr>
<tr>
<td>AR(1) coefficient log($Y_N$)</td>
<td>0.97</td>
<td>0.96</td>
<td>Const and Serv V. Added 1997Q1-2014Q2</td>
</tr>
<tr>
<td>SD log($Y_N$)</td>
<td>1.9%</td>
<td>0.6%</td>
<td>Const and Serv V. Added 1997Q1-2014Q2</td>
</tr>
<tr>
<td>AR(1) coefficient log($Y_T$)</td>
<td>0.87</td>
<td>0.96</td>
<td>Agr and Ind V. Added 1997Q1-2014Q2</td>
</tr>
<tr>
<td>SD log($Y_T$)</td>
<td>4.2%</td>
<td>1.5%</td>
<td>Agr and Ind V. Added 1997Q1-2014Q2</td>
</tr>
</tbody>
</table>

Notes. Spreads and Debt to GDP ratios come from the OECD. Data on tradables and nontradables value added are from the Spanish National Statistics Institute (INE). See Appendix A.2 for a detailed description of data.
value of staying in markets and the value of defaulting. For that reason the model predicts a very weak relationship between the two variables. In line with the model, the data for Spain do not show a clear relationship either.

Figure 4 panel (a) presents the result of the calibration of the exogenous drop in government spending in default ($d_g$). This figure shows the evolution of government spending before, during and after default. The paths are expressed in percentage deviations with respect to real government spending four years before the default episode. The blue line is computed using model simulated data, the black line corresponds to the average path computed using real data from historical default episodes, whereas the grey shaded area shows a data-implied two standard deviation confidence interval. As shown in the figure, the two government spending series are similar and the model implied path (dashed blue) is within the data confidence interval. Even though the fall during the default year is not the same, the government spending levels before and after the default event are closely matched.

I also use this type of exercise to check whether the parameters of the productivity cost of default ($d_0$ and $d_1$), which are calibrated using data on spreads, are sensible in terms of GDP costs during default. To do so I compare the evolution of GDP during historical default episodes with the model-implied average path. This exercise is shown in 4 panel (b). The figure depicts similar data and model implied paths, suggesting that calibrated productivity costs are reasonable.

8 Results

In this section I use the model to quantify the effects of fiscal austerity in Spain. I define austerity as the negative government spending shocks $\epsilon_t^G$ that hit the economy after the second quarter of 2010. The exercise simply consists of running a counterfactual and test what would have happened if these negative shocks on $G$ were absent after 2010-Q2. I follow two steps to perform the exercise. First, I recover the realization of shocks such that model implied paths for tradable output ($Y_T$), nontradable output ($Y_N$), sovereign spreads and government consumption ($G$) best fit the empirical paths. Second, I run a counterfactual without austerity shocks, and compare implied paths of endogenous variables.
Getting Smoothed Shocks. As noted in section 7, the model does a good job matching relevant moments for Spain. As it is usually done in the DSGE literature, I can use this structure to extract the model implied shocks that hit the economy during and after the last financial crisis. To do so I employ a Particle Smoother as in Fernandez-Villaverde and Rubio-Ramirez (2007). Consider the following state space representation,

\begin{align}
S_t &= f(S_{t-1}, \epsilon_t) \\
Y_t &= g(S_t) + \eta_t
\end{align} \tag{34} \tag{35}

Equation (34) is the state equation, where \( f(\cdot) \) is a nonlinear function of the state variables in the previous period \( S_t \), and \( \epsilon_t \) is a vector of structural shocks. Moreover, equation (35) represents the measurement equation where \( g(\cdot) \) is a nonlinear function of the states, and \( \eta_t \) is a vector of measurement errors.

The observables I use to perform the filtering exercise are: tradable output \( (Y_T) \), nontradable output \( (Y_N) \), government consumption \( (G) \) and sovereign spreads \( (s) \). The data correspond to the period 1997Q1-2014Q2. I calibrate the measurement error variances and set them equal to 10% of the variance of each data series.\(^{16}\) Three of the four structural shock processes are unobserved: the risk aversion shock and both productivity shocks. Government expenditure shocks are directly observed from data.

Note that the smoothed productivity shocks computed in this exercise should be interpreted as any private sector shock affecting firms marginal costs. The list of factors that might affect marginal costs includes productivity levels, but also financial costs and any shock affecting capital stock levels. I will refer to them in this section as “fundamental shocks”.

Figure 5 presents the data series and the smoothed variables implied by the model. The figure shows that the model can actually replicate the recent evolution of macro variables in Spain. Indeed, even though I am including measurement errors, the true series are close to the model implied ones.

Figure 6 shows the smoothed exogenous states computed by the smoother: nontradable and

\(^{16}\)I use 10,000 particles for the results.
tradable productivity, government spending and risk premium. The model suggests that the most important driver of the financial crisis has to do with fundamentals in the nontradable sector. The drop in $A_N$ is almost ten percent during 2009. In turn, the tradable sector productivity level is above its mean most of the sample and recovers quickly to pre-crisis levels during 2009. The European debt crisis that started in 2011 showed a drop in productivity in both sectors, but of lower magnitude compared to the financial crisis. In the debt crisis the government reduced government spending quickly. From 2011Q3 to 2013Q1 detrended government spending dropped by 10%. Risk aversion shows an upward trend at the beginning of the debt crisis. Interestingly, it starts falling from 2012, year in which the Outright Monetary Transactions (OMT) program was announced. This evolution of the risk aversion reflects the fact that the debt crisis was also affected by external factors.

Quantifying the Effects of Austerity. Having a calibrated model allows me to run a counterfactual exercise to isolate the impact of fiscal austerity measures. This exercise consists of simulating the model using the smoothed shocks but setting the austerity shocks to zero after the second quarter of 2010, time in which the main austerity measures were announced. In particular, if $\epsilon^g \equiv \left\{ \epsilon^g_{97Q1}, \epsilon^g_{97Q2}, \ldots, \epsilon^g_{10Q2} \right\}$ is the vector of smoothed $G$ shocks from 1997Q1 to 2014Q2, the counterfactual “no austerity” sequence of shocks is $\tilde{\epsilon}^g \equiv \left\{ \epsilon^g_{97Q1}, \epsilon^g_{97Q2}, \ldots, \epsilon^g_{10Q2}, 0, 0, \ldots, 0 \right\}$.

Figure 7 presents the counterfactual paths for government consumption, GDP, consumption and hours worked. Under the counterfactual, $G$ slowly converges to steady state levels instead of dropping steeply from the second half of 2010. Relative to this counterfactual, figure 7 shows that there is an important drop in real GDP. In particular, as a consequence of austerity measures real GDP is 4.5% lower during the first half of 2014. This implies an annual growth rate 1.1% lower from 2010Q2. Actually, there is a similar impact on private consumption levels. This drop in economic activity is explained by a drop in ours worked due to the downward nominal wage rigidity in the labor market.

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17One key driver of the Spanish crisis has to do with the banking sector and credit availability. These factors are not explicitly modeled, but their effects on firms marginal costs are included in the smoothed private sector fundamentals $A_N$ and $A_T$.

18Bocola and Dovis (2016) assess the importance of fundamentals in the last European debt crisis.

19The announced measures included cuts in public wages and public investment, reductions in public health related expenses. See https://www.ft.com/content/91ca42de-5d9e-11df-b4fc-00144feab49a
Figure 8 shows the impact of austerity on sovereign spreads and debt to GDP levels. The results indicate that austerity was not effective in reducing debt to GDP levels. In fact, there is not a significant change in the path of this variable after 2010. Interestingly, sovereign spreads increase as a consequence of austerity. Spreads would have been two percentage points lower after 2011 if austerity had not been implemented. I will provide more details on this below, but notice that this results has to do with the fact that austerity decreased the value of staying in markets, whereas it increased the value of defaulting.

The result that spreads increased might be surprising given that the main objective of implementing fiscal consolidation is to reduce debt levels. However, a steep reduction in government spending has other effects in real variables that might have changed spreads. In particular, the total impact depends on what happens with the value of repaying $V^{\mathcal{R}}(S)$ and the value of default $V^{\mathcal{D}}(S)$. These two value functions depend on the state vector $S$. Indeed, three of the six state variables change with austerity: sovereign debt ($B$), lagged wages ($w_{-1}$) and government spending ($G$). Therefore, there are three different channels through which austerity impact the value functions and, hence, sovereign spreads. Figure 9 shows the impact of each channel taken separately. In this figure I start from the counterfactual scenario (black line, no austerity) and plug each channel individually to check its sign and size.

The red line in figure 9 represents the effect of austerity through debt. Spreads fall because of this channel. The intuition is that austerity measures reduce the stock of debt that needs to be repaid, which increases the value of staying in markets. For a given a value of defaulting, this ends up reducing default risk. The second channel happens through the other endogenous state, lagged wages. Lower government spending decreases the path of wages, which actually reduces the cost of default. When the country defaults there is an important drop in productivity levels which generates a reduction in demand for labor, increasing unemployment. The lower wages are, the less important is this increase in unemployment at the default event. Hence, lower wages imply a higher value of defaulting and this translates into higher spreads.

The last effect has to do with the direct impact of government spending on consumption levels and hours worked, and therefore on the instantaneous utility level $u(C, H)$. The channel is depicted in green in figure 9. A lower $G$ reduces the value of repaying $V^{\mathcal{R}}(S)$ and increases the
value of default $V^D(S)$. It lowers $V^R(S)$ because a drop in $G$ reduces output and consumption due to high fiscal multipliers. Lower consumption implies a lower instantaneous utility and hence a lower value of repaying. On the other hand, a lower $G$ increases $V^D(S)$ because, in financial autarky, a lower $G$ is effectively a lower tax level $T$.\footnote{Remember that the government budget constraint in autarky is simply $p_cG = T$.} This lower tax level allows consumption in default to be higher, and hence it increases the value of default. As we can see in figure 9, this last channel seems to be the most important one.

Fiscal austerity reduces the stock of debt. However, due to the downward nominal wage rigidity, there are other negative real effects associated with austerity that end up increasing sovereign spreads even with a lower debt level.

**Wages and Employment.** One concern regarding the results from this section might be that I am overstating the importance of downward nominal wage rigidity and, as a result, the model generates an excessive drop in hours worked after 2010. This is a reasonable concern given the fact that I do not use nominal wages or employment to calibrate the model or extract model implied structural shocks.

Figure 10 compares data and model implied series for hours and nominal wages. As shown in panel (a), model implied wages seem to fall more than nominal wages in the data. This suggests that the downward nominal wage parameter is appropriate. Panel (b) shows the annual percentage change in hours worked from 1997 to 2013. The two series are of course different but both of them show important drops in hours after 2010. Particularly, the model shows an average drop in hours of 3.9% after 2010, whereas the data details an average decrease of 3.14%.

### 9 Conclusions

I proposed a small open economy model with strategic sovereign default to analyze the effects of fiscal austerity. The results indicate that austerity had an important impact on output, consumption and employment. Moreover, austerity was not effective in reducing debt to GDP ratios and it actually increased sovereign spreads.

This paper contributes to the general discussion about the effects of austerity measures in
different ways. First, it proposes a model with a novel feature: the combination of realistic fiscal rules with strategic sovereign default. This model allows me to run counterfactuals and analyze implications of different policies. Second, it highlights the importance of labor market frictions as an important propagation mechanism. Wage rigidity not only affects the size of fiscal multipliers, but also affect the total impact on sovereign spreads. Third, it emphasizes the fact that the effect of austerity measures is strongly state dependent. In fact, in some situations it might actually increase default risk instead of reducing it.


A Appendix

A.1 Implementable Equilibrium Equations

\[(H^s_t)^\theta (C_t)^\sigma = \frac{w_t}{p_{Ct}}\]  
\[p_{Ct} = \left[\omega^\mu p_{Nt}^{1-\mu} + (1 - \omega)^\mu]\right]^{\frac{1}{1-\mu}}\]  
\[p_{Nt} = \frac{\omega}{1 - \omega} \left(\frac{C_{Tt}}{C_{Nt}}\right)^{1/\mu}\]

\[\alpha_T A_{Tt} (H^d_{Tt})^{\alpha_T - 1} = w_t\]  
\[\alpha_N p_{Nt} A_{Nt} (H^d_{Nt})^{\alpha_N - 1} = w_t\]

\[Y_{Nt} = A_{Nt} (H^d_{Nt})^{\alpha_N}\]  
\[Y_{Tt} = A_{Tt} (H^d_{Tt})^{\alpha_T}\]  
\[G_{Nt} + C_{Nt} = Y_{Nt}\]  
\[C_{Tt} + G_{Nt} + \delta B_t = Y_{Tt} + q_t [B_{t+1} - (1 - \delta)B_t]\]  
\[(w_t - \gamma w_{t-1})(H^s_t - H^d_{Nt} - H^d_{Tt}) = 0\]  
\[w_t \geq \gamma w_{t-1}\]
A.2 Data

- **Value added in nontradable and tradable sectors** come from the Spanish Statistical Office (INE). I define the nontradable sector as Construction and Services, the rest of the economy (Industry and Agriculture) is considered as tradable. Data were linearly detrended.\(^{21}\)

- **Government spending** \(G\) is defined as government consumption and investment. Data from from the OECD Economic Outlook Number 99 (2016).\(^{22}\) These data are annual, but I interpolate them to quarterly frequency using Chow and Lin (1971) method.

- **Net taxes** \(T\) are defined as taxes and social security contributions minus social security benefits paid by the general government. Data from from the OECD Economic Outlook Number 99 (2016).

- **Public debt** \(B\) and **sovereign spreads** also come from OECD Economic Outlook Number 99 (2016). Debt corresponds to general government gross financial liabilities. Spreads are defined as the difference of 10 year bond yields with respect to a 10 year German bond.

- **Sovereign debt average maturity** data come from the Spanish Treasury.\(^{23}\)

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\(^{21}\)See: [http://www.ine.es/jaxiT3/Tabla.htm?t=9148&L=0](http://www.ine.es/jaxiT3/Tabla.htm?t=9148&L=0)


\(^{23}\)See [http://www.tesoro.es/sites/default/files/estadisticas/02I.pdf](http://www.tesoro.es/sites/default/files/estadisticas/02I.pdf)
A.3 Solution Method

I solve the model using global numerical methods. I use Chebychev polynomials with a Smolyak sparse grid defined using Judd et al. (2014) method. Given that the model has an occasionally binding constraint, I approximate the solution of the model using two separate polynomials, one when the constraint binds and another one it does not. Let \( S = \{ B, w_1, A_N, A_T, G, m \} \) be a the states vector, and \( \iota(S) \) be an indicator that is equal to one if the constraint binds and zero otherwise. Hence, if \( w^*(S) \) is the wage level that clears the labor market,

\[
\iota(S) = \begin{cases} 
1 & w^*(S) \geq w_1 \\
0 & w^*(S) < w_1 
\end{cases}
\]

Any equilibrium object \( x(S) \) is approximated doing the following piece-wise approximation,

\[
x(S) = \iota(S)\gamma_1^{x'}T(S) + (1 - \iota(S))\gamma_0^{x'}T(S)
\]

Where \( \gamma_1^{x'} \) and \( \gamma_0^{x'} \) are vectors of coefficients and \( T(S) \) is a vector of Chebychev’s polynomials. I index the numerical solution by \( \{ [\gamma_1^R, \gamma_0^R], [\gamma_1^D, \gamma_0^D], q \} \), where \( [\gamma_1^R, \gamma_0^R] \) and \( [\gamma_1^D, \gamma_0^D] \) are the Chebychev’s coefficients for the value of repaying \( V^R \) and the value of defaulting \( V^D \). The steps for the numerical solution are the following:

**Step 0 (a).** Define grids for endogenous and exogenous states. Define Chebychev’s polynomials for the chosen grid points \( T(S) \). Set an initial guess for value functions \( V^R_{(0)}(S) \) and \( V^D_{(0)}(S) \) and price of debt \( q_{(0)} \). Compute Chebychev’s coefficients for each one of these functions.

**Step 0 (b).** Compute private equilibrium allocations for each one of the grid points defined in the previous step. There are two different cases, one when the economy has access to markets and another when the economy is in default. In this second case, incorporate the productivity costs associated with default. In particular compute consumption \( C(S, d) \), hours \( H(S, d) \), price of government spending \( p_G(S, d) \) and output \( Y(S, d) \). These allocations are a function of the states \( S \) and the default decision \( d \) (\( d = 1 \) if default, \( d = 0 \) otherwise).

**Step 1.** Update the value functions using the private sector allocations, price schedule
and value functions from previous step. Hence, for iteration $i$ the updated value function are computed by,

1. Define the value before deciding whether to default or not as,

$$V_{(i-1)}(S) = \left\{ V^R_{(i-1)}(S); V^D_{(i-1)} \left( (1 - \psi)B, w_{-1}, A_N, A_T, (1 - d_g)G, m \right) \right\}$$ (46)

2. Update value of repaying

$$V^R_{(i)}(S) = u \left( C(S, 0), H(S, 0) \right) + \beta E \left\{ V_{(i-1)}(S')|S \right\}$$ (47)

where the low of motion of the endogenous states is given by,

$$p_G(S, 0)G + \delta B = t^*Y(S, 0) + \frac{\gamma T}{4}B + q_{(i-1)}(S_1, S_2) \left[ B' - (1 - \delta)B \right]$$

$$w'_{-1} = w(S, 0)$$

where $S_1 = \{ B, w_{-1} \}$ and $S_2 = \{ A_N, A_T, G, m \}$. When getting $B'$ from its law of motion, get the value of $B'$ is on the left side of the debt Laffer curve. This is done to rule out one possible source of multiplicity.

3. Value of defaulting

$$V^D_{(i)}(S) = u \left( C(S, 1), H(S, 1) \right) + \beta E \left\{ \phi V_{(i-1)}(S') + (1 - \phi) V^D_{(i-1)}(S')|S \right\}$$ (48)

Where the budget constraint of the government and law of motion of endogenous states are,

$$T(S, 1) = p_G(S, 1)G$$

$$B' = BR^*$$

$$w'_{-1} = w(S, 1)$$
Step 2. With the updated value functions from the previous step redefine the default decision,

\[ d(S) = \left\{ \begin{array}{ll} 1 & V^D_{(i)}(S) \geq V^R_{(i)}(S) \\ 0 & V^D_{(i)}(S) < V^R_{(i)}(S) \end{array} \right. \]

Then update the price of debt using the following expression,

\[ \tilde{q}(S) = \tilde{\beta}E \{ m' \{ (1 - d(S')) (\delta + (1 - \delta)q(S')_{(i-1)}) \} + d(S')(1 - \psi) \} | S \}

Step 3. compute distances \( r^d = \left| \tilde{q}(S) - q(S)_{(i-1)} \right| , r^R = \left| V^R_{(i)}(S) - V^R_{(i-1)}(S) \right| \) and \( r^D = \left| V^D_{(i)}(S) - V^D_{(i-1)}(S) \right| \). Check if they are lower than a tolerance value, if not, update prices and value functions and go to step 1 (set \( i = i + 1 \)). When updating the price function use a dampening parameter \( \theta \), such that

\[ q(S)_{(i)} = \theta \tilde{q}(S) + (1 - \theta)q(S)_{(i-1)} \]

Details. I stop the algorithm when \( r^d < 10^{-3} \) and \( r^R, r^D < 10^{-5} \). The dampening parameter is set to \( \theta = 0.3 \). The grids for the states \( S = \{ B, w, A_N, A_T, G, m \} \) are set with approximation levels \( \mu = \{ 5, 5, 2, 2, 2, 2 \} \). Grids limits are determine as follows. Minimum debt is \( 0.6\bar{B} \) and maximum \( 1.7\bar{B} \), where \( \bar{B} \) is set to match a steady state debt to GDP ratio of 70%. Minimum and maximum lagged wages are respectively \( 0.6\bar{w} \) and \( 1.4\bar{w} \), where \( \bar{w} \) is the steady state wage. Limits for productivity levels and risk premium grids are set to 5 standard deviations below and above the mean. Given the important variation in government spending during the period of analysis, I set the grid for government spending 10 standard deviations below and above the mean. I am suing Gauss Hermite to compute integrals, with five nodes for each exogenous state.
Figure 1: Tax Rules in Eurozone

Notes. Annual data up to 2007, starting points are around 1970 (they vary depending on each country). Net Taxes are total tax income minus transfers. Debt is defined as general government gross financial liabilities (see Appendix A.2 for data sources).
Figure 3: Spreads and Fundamentals

Notes. The figures show the relationship between spreads and (a) GDP, (b) demeaned debt to GDP ratio, and (c) Government Consumption. The solid line is estimated by fitting a fourth degree polynomial to model implied data (simulation of 10,000 periods). The dots highlight true data points for Spain (1995Q1-2014Q2). Blue dots are observations before 2012, red dots highlight post 2012 data points.
Figure 4: Default events: model vs data

Notes. Figures show the evolution of Government Consumption (Panel a) and GDP (Panel b) in typical default episodes. Units are percentage deviation with respect to four years before the default event. Black lines represent data averages, the grey shaded area correspond to 2 standard deviations interval around the data implied average, and blue lines come from model simulations. Data series are average paths for recent default episodes taken from Schmitt-Grohé and Uribe (2016b). Data on GDP and government consumption come from World Bank’s World Development Indicators. Model implied series come from 10,000 simulations from the calibrated model.
Figure 5: Smoothed Series and Data

Notes. This figure shows the data employed in the filtering exercise. Red dashed lines are data series, solid black lines are smoothed series from the model. Government consumption, tradable and nontradable output are log detrended values. Spreads are annual and expressed in percentages.
Figure 6: Smoothed States

Notes. This figure shows the smoothed states computed using the Particle Smoother. The solid black line is the mean value, the grey bars highlight Eurozone recessions.
Figure 7: Effects of Fiscal Austerity

Notes. Black line shows smoothed variables with austerity shocks after 2010Q2. The shaded area highlights two standard deviation intervals around the austerity case. Red line depicts a counterfactual exercise where austerity shocks are set to zero after 2010Q2.
Figure 8: Effects of Fiscal Austerity

Notes. Black line shows smoothed variables with austerity shocks after 2010Q2. The shaded area highlights two standard deviation intervals around the austerity case. Red line depicts a counterfactual exercise where austerity shocks are set to zero after 2010Q2.
Figure 9: Decomposing Impact on Spreads

Spreads

Value in markets

Value in default

Notes. Black line represents the counterfactual scenario without austerity measures after 2010-Q2. Color lines show the impact of austerity through three different channels: reduction in debt levels (red), lower wages (blue) and direct impact on consumption, hours and instantaneous utility level $u(C, H)$ (green).
Figure 10: Wages and Hours Worked

Notes. Black lines are model implied series, dashed blue lines indicate data. Panel (a) shows nominal wages series. Data implied nominal wages are linearly detrended to be consistent with the fact that the series used in the particle smoother are also detrended. Panel (b) shows data and model implied annual percentage change in hours worked. Real data on hours come from OECD economic outlook Number 99.
References


